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D0-branes as light-front confined quarks

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Abstract. We argue that different aspects of light-front QCD at the confined phase can be recovered by the matrix quantum mechanics of D0-branes. The relevant matrix quantum mechanics is obtained from dimensional reduction of pure Yang–Mills theory to dimension $0+1$. The aspects of QCD dynamics which are studied in correspondence with D0-branes are: (1) phenomenological inter-quark potentials, (2) the whiteness of hadrons and (3) scattering amplitudes. In addition, some other issues such as the large-N behavior, the gravity–gauge theory relation and also a possible justification for involving "non-commutative coordinates" in the study of QCD bound states are discussed.

1 Introduction

The idea of a string theoretic description of gauge theories is an old one [1,2]. Despite the years that passed since this idea was launched, it is still actively developed in different research works in theoretical physics [3–6]. On the other hand, in the last years our understanding of string theory has changed dramatically; a series of events which is usually called the "second string revolution" [7]. The aim of this is to formulate a unified string theory as a fundamental theory of the known interactions. One of the best applicable tools in the above program are Dp-branes [8,9]. It is conjectured that Dp-branes are a perturbative representation of non-perturbative (strongly coupled) string theories.

It has been known for a long time that hadron–hadron scattering processes have two different behaviors depending on the amount of momentum transfer [10,11]. At large momentum transfer interactions appear as interactions between the hadron constituents, partons or quarks, and some qualitative similarities to electron–hadron scattering emerge. At high energies and small momentum transfers Regge trajectories are exchanged. Regge trajectories provide a motivation for the first string-based picture of the strong interactions. However, the good fitting between the Regge trajectories and the mass of strong bound states is yet unexplained [1,12].

Deducing the apparently different observations discussed above from a unified picture is the challenge of present day theoretical physics, and it is tempting to search the application of the recent string theoretic progresses in this area. In this way one may find the Dp-branes good tools whose dynamics may be taken as a proper effective theory for the bound states of quarks and QCD strings (QCD electric fluxes). To use the string theory tools for QCD strings one should replace the string theory parameters by those of QCD in a proper way. The case here is in the reverse direction compared to the early days of string theory as the theory of the strong interaction, to string theory as the theory of gravity.

To push further the above idea, in two works [13,14], taking the dynamics of D0-branes as a toy model, the potential and the scattering amplitude of two D0-branes were calculated. It is found that the potential between static D0-branes is a linear potential [15–18]. Also the potential between two fast decaying D0-branes, which in the extreme limit see each other instantaneously, has been calculated and the general results are found to be in agreement with phenomenology [15,16,18]. The scattering amplitude of two D0-branes was calculated in [14]. Based on the results of [19], it is shown that the cross section obeys the Regge pole expansion. Regge behavior has been used some years ago to fit the hadron–hadron total cross section data successfully [20,21] (see also [22–25] for some more recent applications of this behavior).

Based on the results of [13,14] and after some further discussions, we argue that different aspects of the light-front formulation of QCD may be recovered by the matrix quantum mechanics of D0-branes. In this paper, we consider the matrix quantum mechanics resulting from dimensional reduction of $d+1$ dimensional pure $U(N)$ YM theory to dimension $0+1$. The detailed procedure of constructing this matrix mechanics is presented in [26]. In analogy with string theory ($d = 9$ or 25), we call *D0-branes* the free-particles sector of the moduli space. We hope that these kinds of studies will shed light on the possible new relation between D-brane dynamics and gauge theories.

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Also we adjust our discussions so as to be in reasonable contact with the known phenomenological aspects, though an exact match with experiments should not be expected at this level.

In Sect. 2 we review the distinguished role of light-front coordinates for explaining the scaling behavior of hadrons structure functions: the same behavior as is taken as the consequence of point-like substructure in hadrons. In Sect. 3 a short review of matrix quantum mechanics of D0-branes is presented. In Sect. 4 the calculation of the inter-D0-branes potential will be presented. A discussion of the "whiteness" of D0-branes bound states is given in Sect. 5. In Sect. 6 we deal with the problem of scattering. Section 7 is devoted to a discussion. Three issues are discussed in Sect. 7: (1) the large-N limit, (2) quarks, gauge theory and gravity and the relation between solutions, and (3) non-commutativity. The discussion of the non-commutativity concerns a possible justification for the appearance of "non-commutative" coordinates in the study of "non-Abelian" bound states, such as bound states of quarks and gluons.

2 QCD, light cone and constituent quark picture

Before the gauge theoretic description of the strong interaction, QCD, there was the constituent quark model (CQM) for hadrons. According to CQM a meson is just a quark–antiquark bound state and a baryon is a three-quark one. The bound-state problem has been extensively studied for years by the phenomenological inter-quark potentials to calculate various low-energy quantities. The agreement between calculated and observed quantities has always been well enough to justify pursuing this approach to study the hadron properties [15].

Presently QCD has been established to be the underlying theory for strong bound states and also it has been understood that the QCD vacuum is a very complicated medium. In low energy the coupling constant is large, so quantum fluctuations are highly excited. This means that basically the "sea" of quarks and gluons makes a considerable contribution to the properties of hadrons. Moreover, phenomena like confinement are believed to be direct consequences of the complex nature of the QCD vacuum. So it seems that the hadron picture of QCD is not reconcilable with any few-body picture of the hadrons, like CQM (see [27] for a good discussion on this point).

Experimentally, the substructure of hadrons is probed in sufficiently large momentum transfer scatterings of a fundamental particle, e.g. an electron, in the so-called deep inelastic scattering (DIS) experiments. The existence of a point-like substructure, a parton or quark, is taken as the reason for the "scaling" behavior of the hadron structure functions, i.e. the absence of any "scale" is the consequence of point-like objects [10]. Along Bjorken's argument, and as we shall recall below, this scaling behavior has a simple interpretation from the light cone point of view of the processes which are involved in DIS. The story is the same for Feynman's parton picture of a DIS experiment and the light cone frame's cousin, the infinite momentum frame (IMF) [28]. By this simple interpretation of scaling in the light cone frame we hopefully have a constituent picture for hadrons reconcilable with QCD, and this is the reason for developing the light cone formulation of QCD during the past years [27,29,30].

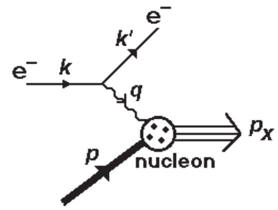

Fig. 1. The lowest order process of a DIS experiment

The unpolarized cross section of DIS in the lowest order is given by[1]

$$k'_0 \frac{\mathrm{d}\sigma}{\mathrm{d}^3k'} = \frac{2M}{s-M^2}\frac{\alpha^2}{Q^4} l_{\mu\nu}W^{\mu\nu}, \tag{2.1}$$

with

$$W_{\mu\nu}(p,q) = \frac{1}{4M}\sum_\sigma \int \frac{\mathrm{d}^4y}{2\pi}\mathrm{e}^{\mathrm{i}q\cdot y}\langle p,\sigma|[J_\mu(y),J_\nu(0)]|p,\sigma\rangle, \tag{2.2}$$

$$l_{\mu\nu} = 2\left(k_\mu k'_\nu + k_\nu k'_\mu - \frac{1}{2}Q^2\eta_{\mu\nu}\right), \quad q = k - k',$$
$$q^2 = -Q^2 < 0. \tag{2.3}$$

M and s are the mass of the nucleon and total energy respectively. The momenta are specified in Fig. 1. Also, we define the useful parameters

$$\nu = \frac{p\cdot q}{M}, \quad x = \frac{Q^2}{2M\nu}, \quad y = \frac{2M\nu}{s-M^2}. \tag{2.4}$$

Note that the parameters x and y are dimensionless. In the rest frame of the nucleon (target) we choose the z-axis to be along the virtual photon momentum; then we have

$$p = (M,0,0,0), \quad q = (\nu,0,0,-\sqrt{\nu^2+Q^2}). \tag{2.5}$$

In the so-called Bjorken limit, $Q^2\to\infty$, $\nu\to\infty$ and $x=$ fixed, we have $q=(\nu,0,0,-\nu-Mx)$. Now the statement of Bjorken scaling is as follows: Up to a kinematical coefficient, the hadronic tensor $W_{\mu\nu}$ depends only on the parameter x and not on Q^2. To see this, it is convenient to use light cone variables $a^\pm = (a^0 \pm a^3)/2^{1/2}$ with scalar product $a\cdot b = a^+b^- + a^-b^+ - a_\mathrm{T}\cdot b_\mathrm{T}$. Thus one writes

$$W_{\mu\nu} \sim \frac{1}{4M}\int \mathrm{d}y^-\mathrm{e}^{\mathrm{i}q^+y^-}\int \mathrm{d}y^+\mathrm{d}^2y_\mathrm{T}\mathrm{e}^{\mathrm{i}q^-y^+}$$
$$\times \langle p|J_\mu(y)J_\nu(0)|p\rangle. \tag{2.6}$$

In the Bjorken limit we have

[1] This discussion is borrowed from [11] and [31]

$$q^+ \to -Mx/\sqrt{2} = \text{fixed},$$
$$q^- = (2\nu + Mx)/\sqrt{2} \to \sqrt{2}\nu \to \infty. \quad (2.7)$$

In this limit the integrand of (2.6) contains the rapidly oscillating factor $\exp(\mathrm{i}q^- y^+)$ which kills all contributions to the integral except for those where the integrand is singular. Indeed the singularity of the integrand comes from the current product at $y^+ \sim 0$. In addition, due to causality the integrand vanishes for $y^2 = 2y^+y^- - y_\mathrm{T}^2 < 0$. So the dominant part of the integral comes from $y^+ = y_\mathrm{T} = 0$. It explains the Bjorken scaling, i.e., the q^- no longer exists at $y^+ = 0$. Now it is clear that the light-front coordinates play a distinguished role in the understanding of the scaling behavior in DIS experiments. The same result is also correct for Feynman's parton description of DIS and IMF, the experimental realization of light cone frame [28].

3 Matrix quantum mechanics of D0-branes

According to string theory, Dp-branes are p dimensional objects defined as (hyper)surfaces which can trap the ends of strings [9] and therefore it is reasonable to take their dynamics as a proper effective theory for the bound states of quarks and QCD strings (QCD electric fluxes).

One of the most interesting aspects of D-brane dynamics appears in their *coincident limit*. In the case of coinciding N Dp-branes their dynamics is captured by a $U(N)$ YM theory dimensionally reduced to $p+1$ dimensions of the Dp-brane world volume [32,9,33]. In the case of D0-branes, $p=0$, the above dynamics reduces to quantum mechanics of matrices, because time is the only parameter in the world line. A detailed procedure of constructing this matrix mechanics is presented in [26]. The bosonic Lagrangian resulting from the pure YM is [34][2]

$$L = m_0 \mathrm{Tr}\left(\frac{1}{2}D_t X_i^2 + \frac{1}{4(2\pi\alpha')^2}[X_i, X_j]^2\right), \quad (3.1)$$
$$i,j = 1, ..., d, \quad D_t = \partial_t - \mathrm{i}[a_0, \ ],$$

where $1/(2\pi\alpha')$ and $m_0 = (l_\mathrm{s} g_\mathrm{s})^{-1}$ are the string tension and the mass of the D0-branes, respectively ($l_\mathrm{s} = \alpha'^{1/2}$ and g_s are the string length and coupling, respectively). For N D0-branes the X's are in the adjoint representation of $U(N)$ and have the usual expansion $X_i = x_{i(a)} T_{(a)}$, $(a) = 1, ..., N^2$ [3].

The action (3.1) is invariant under the residual gauge symmetry of unreduced YM theory. The transformations are

$$\boldsymbol{X} \to \boldsymbol{X'} = U\boldsymbol{X}U^\dagger,$$
$$a_0 \to a_0' = U a_0 U^\dagger + \mathrm{i}U\partial_t U^\dagger, \quad (3.2)$$

where U is an arbitrary time-dependent $N \times N$ unitary matrix. Under these transformations one can check that

$$D_t\boldsymbol{X} \to D_t'\boldsymbol{X'} = U(D_t\boldsymbol{X})U^\dagger,$$
$$D_t D_t\boldsymbol{X} \to D_t' D_t'\boldsymbol{X'} = U(D_t D_t\boldsymbol{X})U^\dagger. \quad (3.3)$$

[2] Here we take d arbitrary

[3] To avoid confusion we put the group indices in () always

First let us search for D0-branes in the above Lagrangian: For each direction i there are N^2 variables and not N ones as one expects for N particles. However, there is an ansatz for the equations of motion which restricts the $U(N)$ basis to its N dimensional Cartan subalgebra. This ansatz causes the vanishing of the potential and one finds the action of N free particles, namely

$$S = \int \mathrm{d}t \sum_{(a)=1}^{N} \frac{1}{2} m_0 \dot{\boldsymbol{x}}_{(a)}^2. \quad (3.4)$$

In this case the $U(N)$ symmetry is broken to $U(1)^N$ and the interpretation of N remaining variables as the classical (relative) positions of N particles is meaningful. The center of mass of D0-branes is represented by the trace of the X matrices.

In the case of unbroken gauge symmetry the gauge transformations mix the entries of the matrices and the interpretation of the positions for the D0-branes remains obscure [35]. Even in this case the center of mass is meaningful and the ambiguity of the positions only remains for the relative positions of the D0-branes. In the unbroken phase the $N^2 - N$ non-Cartan elements of matrices have a string interpretation; they govern the dynamics of the low lying oscillations of strings stretched between the D0-branes.

The dependences of the energy eigenvalues and the size of the bound states are notable. By the scalings [34]

$$t \to g_\mathrm{s}^{-1/3} t,$$
$$a_0 \to g_\mathrm{s}^{1/3} a_0,$$
$$X \to g_\mathrm{s}^{1/3} X, \quad (3.5)$$

one finds the relevant energy and size scales to be

$$E \sim g_\mathrm{s}^{1/3}/l_\mathrm{s},$$
$$l_{d+2} = g_\mathrm{s}^{1/3} l_\mathrm{s}. \quad (3.6)$$

The length scale l_{d+2} should be the fundamental length scale of the covariant $d+2$ dimensional theory whose light cone formulation is argued to be described by the action (3.1) with the longitudinal momentum as m_0 [19]. So it is natural to assume in our case that l_{d+2} (for $d=2$) is the inverse of the $3+1$ dimensional QCD mass scale, denoted by Λ_QCD [4]. In the weak coupling limit $g_\mathrm{s} \to 0$ ($m_0 \gg l_\mathrm{s}^{-1}$) one finds $l_{d+2} \ll l_\mathrm{s}$ which allows one to treat the bound states of a finite number of D0-branes as point-like objects in the transverse directions of the light cone frame[5], and consequently one finds $m_0 \cdot E \sim 1/l_{d+2}^2$, which shows the invariance under Lorentz transformations of this combination. As we will see in Sect. 6 the masses of the intermediate states in the scattering amplitude appear as l_{d+2}^{-1}.

[4] Due to the light-front interpretation, our Λ_QCD differs from [26]. There $l_\mathrm{s} \sim \alpha'^{1/2}$ is taken as $\Lambda_\mathrm{QCD}^{-1}$

[5] Because we admit the discrete longitudinal momentum, m_0, for finite N, we are dealing with discrete light cone quantization (DLCQ) [36]. We do not emphasize this point later

4 Known potentials

To calculate the effective potential between D0-branes one should find the effective action around a classical configuration. This work can be done by integrating over the quantum fluctuations in a path integral. For the diagonal classical configurations, classical representations of D0-branes, the quantum fluctuations which must be integrated over are the off-diagonal entries. This work is equivalent to integrating over the oscillations of the strings stretched between D0-branes. Because here we deal with a gauge theory, and our interest is the calculation around the classical field configuration, to obtain the effective action, it is convenient to use the background field method [37].

To calculate the effective action we write (3.1) in $d+1$ space-time dimensions in the form (in the units $2\pi\alpha'=1$ and after the Wick rotation $t \to \mathrm{i}t$ and $a_0 \to -\mathrm{i}a_0$)

$$L = m_0 \mathrm{Tr}\left(\frac{1}{4}[X_\mu, X_\nu]^2\right), \quad \mu,\nu = 0,1,...,d,$$
$$X_0 = \mathrm{i}\partial_t + a_0, \quad S = \int L\mathrm{d}t, \tag{4.1}$$

where μ and ν are summed over by the Euclidean metric. The one-loop effective action of (4.1) has been calculated several times (e.g. see the Appendix of [38]) and the result can be expressed as

$$\left(\int \mathrm{d}t\right) V(X_\mu^{\mathrm{cl}}) = \frac{1}{2}\mathrm{Tr}\log(P_\lambda^2\delta_{\mu\nu} - 2\mathrm{i}F_{\mu\nu}) - \mathrm{Tr}\log(P_\lambda^2), \tag{4.2}$$

with

$$P_\mu * \equiv [X_\mu^{\mathrm{cl}}, *], \quad F_{\mu\nu} * \equiv [f_{\mu\nu}, *], \quad f_{\mu\nu} \equiv [X_\mu^{\mathrm{cl}}, X_\nu^{\mathrm{cl}}],$$

and

$$P_\lambda^2 = -\partial_t^2 + \sum_{i=1}^{d} P_i^2, \tag{4.3}$$

with the backgrounds $a_0^{\mathrm{cl}} = 0$. The second term in (4.2) is due to the ghosts associated with gauge symmetry.

4.1 Static potential

Here we calculate the potential between two D0-branes at rest. The classical solution which represents two D0-branes at a distance r can be introduced by

$$X_1^{\mathrm{cl}} = \frac{1}{2}\begin{pmatrix} r & 0 \\ 0 & -r \end{pmatrix}, \quad X_0^{\mathrm{cl}} = \mathrm{i}\partial_t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$a_0^{\mathrm{cl}} = X_i^{\mathrm{cl}} = 0, \quad i = 2,...,d. \tag{4.4}$$

So one finds

$$P_1 = \frac{r}{2} \otimes \Sigma_3, \quad P_0 = \mathrm{i}\partial_t \otimes 1_4, \quad P_i = 0, \quad i = 2,...,d, \tag{4.5}$$

where Σ_3 is the adjoint representation of the third Pauli matrix, $\Sigma_3 * = [\sigma_3, *]$. The eigenvalues of Σ_3 are 0, 0, ± 2.

The operator P_λ^2 is found to be

$$P_\lambda^2 = -\partial_t^2 \otimes 1_4 + \frac{r^2}{4} \otimes \Sigma_3^2, \tag{4.6}$$

which is a harmonic oscillator operator whose frequency, reintroducing α', is $\omega \sim r/\alpha'$. The one-loop effective action can be computed[6].

$$\begin{aligned} V(r) &= \left(\frac{d-1}{2}\right)\mathrm{Tr}\log(P_\lambda^2) \\ &= -2\left(\frac{d-1}{2}\right)\int_0^\infty \frac{\mathrm{d}s}{s}\int_{-\infty}^{\infty}\mathrm{d}k_0\, \mathrm{e}^{-s(k_0^2+r^2)} \\ &\quad + \text{traces independent of } r, \end{aligned} \tag{4.7}$$

where 2 is for the degeneracy in the eigenvalue 4 of Σ_3^2, and k_0 is for the eigenvalues of the operator $i\partial_t$. In writing the second line we have used

$$\ln\left(\frac{u}{v}\right) = \int_0^\infty \frac{\mathrm{d}s}{s}(\mathrm{e}^{-sv} - \mathrm{e}^{-su}).$$

The integrations can be performed and one finds

$$\begin{aligned} V(r) &= -2\left(\frac{d-1}{2}\right)\int_0^\infty \frac{\mathrm{d}s}{s}\left(\frac{\pi}{s}\right)^{1/2}\mathrm{e}^{-sr^2} \\ &= 4\pi\left(\frac{d-1}{2}\right)|r| - \infty \quad (\text{independent of } r). \end{aligned} \tag{4.8}$$

The linear potential is of phenomenological interest; see e.g. [15,17,18]. Also it is the same as the one which is consistent with spin–mass Regge trajectories [15–18]. By restoring α' the potential will be found to be

$$V(r) = 4\pi\left(\frac{d-1}{2}\right)\frac{|r|}{2\pi\alpha'}, \tag{4.9}$$

which has the dimension length^{-1}. By comparison with the Regge model one can make an estimate for α' [16,18]. The above potential can be used to describe an effective theory for the relative dynamics of the D0-branes by

$$S = \int \mathrm{d}t\left(\frac{1}{2}\frac{m_0}{2}\dot{\boldsymbol{r}}^2 - 4\pi\left(\frac{d-1}{2}\right)\frac{|\boldsymbol{r}|}{2\pi\alpha'}\right), \tag{4.10}$$

which in the range of validity of the one-loop approximation, mentioned in the previous footnote, is expected to be applicable. Also by this action one obtains the energy scale as $E \sim \alpha'^{-2/3} m_0^{-1/3} \sim g_{\mathrm{s}}^{1/3}/l_{\mathrm{s}}$, as pointed in (3.6). The above action describes the dynamics in the light cone frame with the longitudinal momentum m_0, and recalling (3.6) we have $p^+p^- \sim m_0 E \sim g_{\mathrm{s}}^{-2/3} l_{\mathrm{s}}^{-2} \sim l_{d+2}^{-2}$.

It is not hard to see that the two D0-brane interaction potential is also true for every pair inside a bound states

[6] The one-loop effective action is a good approximation for $\omega \gg m_0\dot{r}^2$. It gives $rg_{\mathrm{s}} \gg l_{\mathrm{s}}\dot{r}^2$ which for $g_{\mathrm{s}} \to 0$ ($m_0 \gg l_{\mathrm{s}}^{-1}$) is satisfied for large separations and low velocities

of D0-branes. So the effective action for N D0-branes is found to be

$$S = \int \mathrm{d}t \left(\frac{1}{2} m_0 \sum_{(a)=1}^{N} \dot{\boldsymbol{r}}_{(a)}^2 - 4\pi \left(\frac{d-1}{2} \right) \sum_{(a)>(b)=1}^{N} \frac{|\boldsymbol{r}_{(a)} - \boldsymbol{r}_{(b)}|}{2\pi\alpha'} \right). \quad (4.11)$$

In a recent work [39], by taking the linear potential between quarks of a baryonic state in the transverse directions of the light cone frame, the structure functions are obtained with good agreement with the observed ones.

It is useful to relate the parameter $1/\alpha'$ in the potential with gauge theory parameters. To do this we need a string theoretic description of gauge theory, but in the light cone frame. The nearest formulation we know for this is light cone–lattice gauge theory (LClgt) [40]. In LClgt one assumes the time direction and one of the spatial directions, say z, in the continuum limit. The light cone variables are defined as the usual $x^{\pm} \sim t \pm z$. Other spatial directions naturally play the role of transverse directions of the light cone frame, which are assumed to be on a lattice in LClgt. Due to the existence of a continuous time x^+, there exists a Hamiltonian formulation [41] of the lattice gauge theory [42]. The relation between the linear confinement potential and the gauge–lattice parameters is given by [41,40]

$$V(r) \sim \frac{g_{\mathrm{YM}}^2}{a^2} |r|, \quad (4.12)$$

with a the lattice spacing parameter in the transverse directions. Comparing this with (4.9) leads to

$$\frac{1}{\alpha'} \sim \frac{g_{\mathrm{YM}}^2}{a^2}. \quad (4.13)$$

4.2 Fast decaying D0-branes[7]

For two *fast decaying* D0-branes one can again calculate the above potential. This work can be done by inserting for example a Gaussian function for k_0 into (4.7). This work is equivalent to restricting the eigenvalues of the operator $\mathrm{i}\partial_t$. Keeping in mind that the eigenvalues of the operators $(X, \mathrm{i}\partial_t, ...)$ represent the information corresponding to classical values of the D0-branes space-time positions[8], we find

$$V(r) = -2 \left(\frac{d-1}{2} \right) \int_0^{\infty} \frac{\mathrm{d}s}{s} \times \int_{-\infty}^{\infty} \mathrm{d}k_0 \left(\frac{1}{\Delta} \mathrm{e}^{\frac{-k_0^2}{\Delta^2}} \right) \mathrm{e}^{-s(k_0^2 + r^2)} \quad (4.14)$$

$$= -2\sqrt{\pi} \left(\frac{d-1}{2} \right) \int_0^{\infty} \frac{\mathrm{d}s}{s} \frac{\mathrm{e}^{-sr^2}}{\sqrt{s\Delta^2 + 1}}, \quad (4.15)$$

in which we assumed that the D0-branes live around time zero. The last expression is infinite, but one can show that the infinite part is r-independent. One takes

$$\frac{\partial V(r)}{\partial (r^2)} = 2\sqrt{\pi} \left(\frac{d-1}{2} \right) \int_0^{\infty} \frac{\mathrm{d}s \mathrm{e}^{-sr^2}}{\sqrt{s\Delta^2 + 1}}, \quad (4.16)$$

which is finite and so the infinity of $V(r)$ is r-independent. The last integral cannot be calculated exactly, though a numerical comparison with phenomenology is possible. The limit $\Delta \to 0$ can be calculated exactly by recalling the relation

$$\lim_{\Delta \to 0} \left(\frac{1}{\Delta} \mathrm{e}^{-k_0^2/\Delta^2} \right) = \sqrt{\pi} \delta(k_0).$$

Inserting the δ-function in (4.14) one finds

$$V(r) \sim -2 \left(\frac{d-1}{2} \right) \int_0^{\infty} \frac{\mathrm{d}s}{s} \, \mathrm{e}^{-s(r^2)} \sim \ln r, \quad (4.17)$$

the last result is obtained after extracting the r-independent infinity. This result is already consistent with the phenomenology of heavy quarks [18,16], of which we know that their weak decay rates grow with $(\mathrm{mass})^5$. In the extreme limit $\Delta \to 0$, in which the two D0-branes see each other "instantaneously", one can take them as two D(-1)-branes (D-instantons). The dynamics of D(-1)-branes are described by the action (4.1), but instead of taking X_0 as $\mathrm{i}\partial_t$ one takes X_0 as a matrix of which the eigenvalues represent the "instants" at which the D(-1)-branes occur. So the above logarithmic result also could be obtained in the D(-1)-brane calculation by taking a classical solution:

$$X_1^{\mathrm{cl}} = \frac{1}{2} \begin{pmatrix} r & 0 \\ 0 & -r \end{pmatrix}, \quad X_0^{\mathrm{cl}} = \begin{pmatrix} t_0 & 0 \\ 0 & t_0 \end{pmatrix},$$
$$a_0^{\mathrm{cl}} = X_i^{\mathrm{cl}} = 0, \quad i = 2, ..., d, \quad (4.18)$$

which represents two D(-1)-branes appearing at time t_0, at distance r.

A comment is in order: from the phenomenological point of view, it is known that in some cases potentials like r^{ξ}, with $\xi \simeq 0.1$, also have produced good results [18, 16]. This maybe can be included in our intermediate result (4.14) or the logarithmic result by recalling the numerical relation $\ln r \simeq r^{\eta \simeq 0}$, which is valid for a range of r [9].

5 White states

To determine the color of an object its dynamics should be studied in the presence of external fields. For a "white" extended object, the center of mass (c.m.) moves as a free particle in a uniform electric field. Now we want to specify the color of the D0-branes bound states. As we will see, although our formulation for the dynamics of D0-branes in external YM fields seems incomplete, a reasonable statement about "whiteness" of D0-branes bound states can be made.

[7] This subsection was modified based on a crucial comment by the referee of Eur Phys. J. C

[8] The eigenvalues of $\mathrm{i}\partial_t$ here are different from their quantum mechanical analogues, which due to the Schrödinger equation are the energies

[9] One can justify this by the relation $\ln r = \lim_{\eta\to 0} \int \mathrm{d}r r^{-1+\eta} = \lim_{\eta \to 0} r^{\eta}/\eta$

5.1 D0-branes in YM background

In classical electrodynamics besides electromagnetic fields produced by different distributions of charges and currents, we also study the dynamics of a charged particle in regions of space where electromagnetic fields exist. A simple question is: What are the problems arising when one studies chromodynamics in this level?

The main problem arises when one introduces sources and matches chromodynamics with the dynamics of colored objects (for example a colored particle). In the case of electrodynamics there is a simple relation. For example the equation of motion of a charge particle with mass m_0 and charge q is

$$m_0\ddot{\boldsymbol{x}} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{5.1}$$

The concept of gauge invariance at this level is understood as the invariance of the equations of motion under gauge transformations, i.e. field strengths are invariant under gauge transformations. Now, in the case of chromodynamics the right-hand side is a matrix and transforms as an object in the adjoint representation under gauge group transformations:

$$\boldsymbol{E} \to \boldsymbol{E}' = U\boldsymbol{E}U^\dagger, \quad \boldsymbol{B} \to \boldsymbol{B}' = U\boldsymbol{B}U^\dagger. \tag{5.2}$$

Thus the problem mentioned arises. As is well known to string theorists, now we have a good candidate for non-commutative coordinates which are the coordinates of coincident D0-branes. First one may rewrite (5.1) for "matrix" coordinates by

$$m_0\ddot{\boldsymbol{X}} = q(\boldsymbol{E} + \dot{\boldsymbol{X}} \times \boldsymbol{B}), \tag{5.3}$$

but it is not enough to have the correct behavior for the first side under gauge transformations. Here the world-line gauge symmetry (3.2) of the D0-brane dynamics helps us to write the generalized Lorentz equation as[10]

$$m_0 D_t D_t \boldsymbol{X} = q(\boldsymbol{E} + D_t\boldsymbol{X} \times \boldsymbol{B}). \tag{5.4}$$

By recalling the relation (3.3) one observes that both sides have the same behavior under gauge transformations. However, it seems that the picture is not complete yet. First, it is not clear what the Lagrangian formulation is of this problem. Second, the precise meaning of the position dependences of the field strengths should be clarified (the same question can be asked for U, the parameter of the gauge transformation).

Now, the crucial observation is the decoupling of the c.m. dynamics from the non-Abelian parts. This is because of the trace nature of the $U(1)$ and $SU(N)$ parts. As we mentioned earlier the c.m. degree of freedom is described by the $U(1)$ part of $U(N)$ [32]. So the position and the momentum of the c.m. can be obtained by a simple trace [35]:

$$\boldsymbol{x}_{\text{c.m.}} \equiv \frac{1}{N}\text{Tr}\boldsymbol{X}, \quad \boldsymbol{p}_{\text{c.m.}} \equiv \text{Tr}\boldsymbol{P}. \tag{5.5}$$

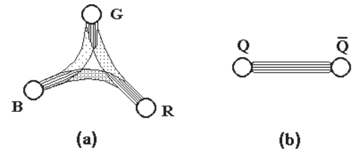

Fig. 2a,b. The net electric flux extracted from each quark is equivalent in a baryon **a** and a meson **b**. The D0-brane–quark correspondence suggests the string-like shape for fluxes inside a baryon **a**

To investigate the kind and amount of the charge of an object its dynamics should be studied in the absence of a magnetic field ($\boldsymbol{B} = 0$) and (for extended objects) in a uniform electric field ($\boldsymbol{E}(x) = \boldsymbol{E}_0$). So the c.m. equation of motion is

$$m_0\ddot{\boldsymbol{x}}_{\text{c.m.}} = q\boldsymbol{E}_{(1)0}, \tag{5.6}$$

where the subscript (1) tells us that the corresponding electric field comes from the $U(1)$ part of $U(N)$. It is understood that the dynamics of the c.m. will not be affected by the non-Abelian part of gauge group. This means that the c.m. is white with respect to $SU(N)$. This behavior of D0-brane bound states is the same as that of hadrons. This means that each D0-brane feels the net effect of the other D0-branes as the white complement of its color. In other words, the field fluxes extracted from one D0-brane to the other ones are the same as the flux between a color and an anti-color, see Fig. 2. As we have shown in Sect. 4, there is a linear potential between each two static D0-branes, which is consistent with this flux-string picture. Also, the number of D0-branes in the bound state, N, equals that of the baryons. As we mentioned before, recently [39] the linear potential between the constituents of baryons, in the transverse directions of the light cone frame, has been used successfully to obtain the structure functions.

As a final note of this part, we recall that the dynamics presented by (5.1) can be taken as for a massless particle in the transverse directions in the light cone frame with longitudinal momentum, $p^+ \equiv m_0$. The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are electromagnetic fields in the transverse directions. We present the derivation of this in the Appendix.

6 Scattering amplitude

As a consequence of asymptotic freedom, in a sudden collision process quarks or partons are assumed to be free. So the probe, an electron or another quark, only interacts with the hadron constituents instead of the hadron as a whole [10,11]. It is the same mechanism as which results in a *scaling* behavior in the hadron structure functions.

Keeping the above in mind it is reasonable to calculate the scattering amplitude between two individual D0-branes, to obtain an impression of the behavior of the

[10] Here we drop the commutator potential in the action of the D0-branes, without any loss of generality. Things may be easier with the symmetrized version of the magnetic part written as $(1/2)(D_t\boldsymbol{X} \times \boldsymbol{B} - \boldsymbol{B} \times D_t\boldsymbol{X})$

scattering amplitude of two hadrons of which D0-branes are assumed as their quarks. Also it is natural to assume that this result is valid for the high energy elastic regime of hadron collisions.

Here we use the result of [19]. In [19] it is shown that the quantum travelling of D0-branes can be understood by the field theory of Feynman graphs and the corresponding amplitudes in the light cone frame. In the following we review this approach to calculate the amplitude.

We concentrate on the limit $\alpha' \to 0$. In this limit, in order to have a finite energy one has

$$[X_i, X_j] = 0, \quad \forall\, i, j, \tag{6.1}$$

and consequently the potential term in the action vanishes. So D0-branes do not interact and the "classical action" reduces to the action of N free particles. We take this classical action also in the quantum case; it is equivalent to the assumption that two quarks in two spatially well separated hadrons do not interact with each other. Since hadrons are white one can rely on this assumption. However, the above observation fails when D0-branes come to each other. When two D0-branes come very near each other, two eigenvalues of the X_i matrices will be equal and the corresponding off-diagonal elements can get non-zero values. This is the same story as gauge symmetry enhancement. The fluctuations of these off-diagonal elements are responsible for the interaction between D0-branes in bound states.

In the coincident limit the dynamics is complicated. The relative matrix position may be taken as

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{r}/2 & \boldsymbol{Y} \\ \boldsymbol{Y}^* & -\boldsymbol{r}/2 \end{pmatrix}, \tag{6.2}$$

where Y^* is the complex conjugate of Y. By inserting this matrix into the Lagrangian one obtains

$$S = \int \mathrm{d}t \frac{1}{2}\Big((2m_0)\dot{\boldsymbol{X}}^2_{\mathrm{c.m.}} + m_0 \dot{\boldsymbol{Y}} \cdot \dot{\boldsymbol{Y}}^* - \frac{m_0}{4}\frac{1}{4(2\pi\alpha')^2} (1-\cos^2\theta)\boldsymbol{r}^2 \boldsymbol{Y}\cdot\boldsymbol{Y}^* + \frac{m_0}{2}\dot{\boldsymbol{r}}^2 + O(Y^3)\Big), \tag{6.3}$$

with $X_{\mathrm{c.m.}}$ the center of mass and θ the angle between $\boldsymbol{r}$ and the complex vector $\boldsymbol{Y}$. As is apparent in the $\alpha' \to 0$ limit, which is the case of our interest, the r element does not take large values and have a small range of variation. In the high-tension approximation of strings ($\alpha' \to 0$), one can take the separation of the D0-branes to be a constant of order $r \sim g_{\mathrm{s}}^{1/3} l_{\mathrm{s}}$. As has been noted in Sect. 3, this length is the typical size of the D0-brane bound states. So

$$S = \int \mathrm{d}t \left(\frac{1}{2}(2m_0)\dot{\boldsymbol{X}}^2_{\mathrm{c.m.}} + \frac{1}{2} m_0 \dot{Y}_\perp \cdot \dot{Y}^*_\perp - \frac{1}{2} m_0 \frac{k^2 r^2}{\alpha'^2} Y_\perp \cdot Y^*_\perp + \frac{1}{2}\frac{m_0}{2}\dot{\boldsymbol{r}} + \cdots\right), \tag{6.4}$$

where in the above k is a numerical factor depending on α' and g_{s}, and $Y_\perp$ is the part of the $\boldsymbol{Y}$ perpendicular to the relative distance $\boldsymbol{r}$. The parallel part of $\boldsymbol{Y}$ behaves as

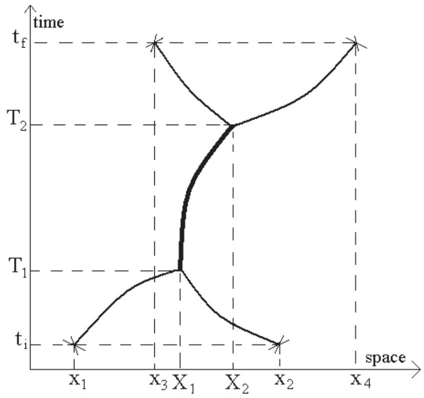

Fig. 3a,b. A typical tree path of D0-branes

a free part. In $d+1$ dimensions of space-time the dimension of $Y_\perp$ is $d-1$, which shows that we have encountered $2 \times (d-1)$ harmonic oscillators, because Y is a complex variable. This is the same number of harmonic oscillators as appears in one-loop calculations (Sect. 4). These harmonic oscillators correspond to vibrations of (oriented) open strings stretched between D0-branes. In the following we ignore the radial momentum and even the angular momentum by dropping the term $m_0 \dot{\boldsymbol{r}}^2$ and set $r = r_0$ for simplicity[11].

For two D0-branes we take the probability amplitude represented by a path integral as

$$\langle x_3, x_4; t_f | x_1, x_2; t_i \rangle = \int \mathrm{e}^{-S}. \tag{6.5}$$

Based on the previous discussion, in the $\alpha' \to 0$ limit for the graph (see Fig. 3) we decompose the path-integral as follows[12]:

$$\begin{aligned} &\langle x_3, x_4; t_f | x_1, x_2; t_i \rangle \\ &= \left[\int \mathrm{e}^{-S}\right]_{\alpha'\to 0} \\ &= \int_{t_i}^{t_f} \mathrm{d}T_1 \mathrm{d}T_2 \int_{-\infty}^{\infty} \mathrm{d}X_1 \mathrm{d}X_2 \\ &\quad \times (K_{m_0}(X_1, T_1; x_1, t_i) K_{m_0}(X_1, T_1; x_2, t_i)) \\ &\quad \times (K_{2m_0}(X_2, T_2; X_1, T_1) \\ &\quad \times K_{\mathrm{oscillator}}(Y_\perp = 0, T_2; Y_\perp = 0, T_1)) \\ &\quad \times (K_{m_0}(x_3, t_f; X_2, T_2) K_{m_0}(x_4, t_f; X_2, T_2)), \end{aligned} \tag{6.6}$$

where $K_m(y_2, t_2; y_1, t_1)$ is the non-relativistic propagator of a free particle with mass m between (y_1, t_1) and (y_2, t_2) and $K_{\mathrm{oscillator}}(Y_\perp = 0, T_2; Y_\perp = 0, T_1)$ is the harmonic oscillator propagator. $\int \mathrm{d}T_1 \mathrm{d}T_2 \mathrm{d}X_1 \mathrm{d}X_2$ is for a summation over different "joining-splitting" times and points. We use in d dimensions the representations

$$K_m(y_2, t_2; y_1, t_1)$$

[11] Setting $r = r_0$ may be justified by a mean value problem in integrations over constant backgrounds in the path integral as $\int r^{d-1} \mathrm{d}r \int DY\, DY^* \mathrm{e}^{-S[r,Y,Y^*]} \sim \int DY\, DY^* \mathrm{e}^{-S[r_0,Y,Y^*]}$

[12] Here similar to field theory we have dropped the disconnected graphs

$$= \theta(t_2 - t_1) \frac{1}{(2\pi)^d} \int \mathrm{d}^d p \mathrm{e}^{\mathrm{i}p\cdot(y_2-y_1) - \frac{\mathrm{i}p^2(t_2-t_1)}{2m}},$$

$$K_{\text{oscillator}}(Y_\perp = 0, T_2; Y_\perp = 0, T_1) = \theta(T_2 - T_1)\left(\frac{m_0\omega}{2\pi\mathrm{i}\sin[\omega(T_2-T_1)]}\right)^{d-1},$$

where $\theta(t_2 - t_1)$ is the step function and ω is the harmonic oscillator frequency, $\omega \sim kr_0/\alpha' \sim kg_\mathrm{s}^{1/3}/l_\mathrm{s}$. Because of the complex nature of $Y_\perp$ the power for the harmonic propagator is $2 \times (d-1)/2$.

All the above results can be translated into the momentum space ($E_k = p_k^2/(2m_0)$ with $k = 1, 2, 3, 4$):

$$\langle p_3, p_4; t_\mathrm{f}|p_1, p_2; t_\mathrm{i}\rangle \sim \mathrm{e}^{\mathrm{i}(E_3+E_4)t_\mathrm{f} - \mathrm{i}(E_1+E_2)t_\mathrm{i}} \times \int \prod_{a=1}^{4} \mathrm{d}x_a \mathrm{e}^{\mathrm{i}(p_1x_1+p_2x_2-p_3x_3-p_4x_4)} \times \langle x_3, x_4; t_\mathrm{f}|x_1, x_2; t_\mathrm{i}\rangle. \quad (6.7)$$

This representation is useful for calculating the cross section. The integrals can be performed and we find

$$\langle p_3, p_4; t_\mathrm{f}|p_1, p_2; t_\mathrm{i}\rangle \sim \delta^{(d)}(p_1+p_2-p_3-p_4)\int_{t_\mathrm{i}}^{t_\mathrm{f}} \mathrm{d}T_1\mathrm{d}T_2\theta(T_2-T_1) \times \exp\left(\frac{-\mathrm{i}(p_1^2+p_2^2)T_1}{2m_0}\right)\exp\left(\frac{-\mathrm{i}q^2(T_2-T_1)}{4m_0}\right) \times \exp\left(\frac{\mathrm{i}(p_3^2+p_4^2)T_2}{2m_0}\right) \times K_{\text{oscillator}}(Y_\perp = 0, T_2; Y_\perp = 0, T_1), \quad (6.8)$$

where $\boldsymbol{q} = \boldsymbol{p}_1 + \boldsymbol{p}_2 = \boldsymbol{p}_3 + \boldsymbol{p}_4$.

To have a real scattering process let us assume

$$t_\mathrm{i} \to -\infty, t_\mathrm{f} \to \infty.$$

We put $T \equiv T_2 - T_1$ which has the range $0 \le T \le \infty$. The integrals yield

$$\langle p_3, p_4; \infty|p_1, p_2; -\infty\rangle \sim \delta^{(d)}(p_1+p_2-p_3-p_4) \times\delta\left(\frac{p_1^2}{2m_0}+\frac{p_2^2}{2m_0}-\frac{p_3^2}{2m_0}-\frac{p_4^2}{2m_0}\right) \int_0^\infty \mathrm{d}T\mathrm{e}^{(-\mathrm{i}T)/(4m_0)(q^2-2(p_1^2+p_2^2))}\left(\frac{m_0\omega}{\sin(\omega T)}\right)^{d-1}. \quad (6.9)$$

Recalling the energy-momentum relation in the light cone gauge [19],

$$2(p_1^2+p_2^2) - \boldsymbol{q}^2 = 2(2m_0)\left(\frac{p_1^2+p_2^2}{2m_0}\right) - \boldsymbol{q}^2 = 2q^+q^- - \boldsymbol{q}^2 = q_\mu q^\mu \equiv q_\mu^2,$$

we find

$$\langle p_3, p_4, E_3, E_4; \infty|p_1, p_2, E_1, E_2; -\infty\rangle \sim \delta^{(d)}(p_1+p_2-p_3-p_4)\delta(E_1+E_2-E_3-E_4) \int_0^\infty \mathrm{d}T\mathrm{e}^{(-q_\mu^2/(4m_0))T}\left(\frac{m_0\omega}{\sin(\omega T)}\right)^{d-1}. \quad (6.10)$$

We perform a cut-off for T for small values as $0 < \epsilon \le T \le \infty$, with ϵ small[13]. By changing the integral variables, $\mathrm{e}^{-2\omega T} = \eta$, we have

$$\langle p_3^\mu, p_4^\mu; \infty|p_1^\mu, p_2^\mu; -\infty\rangle \sim \delta^{(d)}(p_1+p_2-p_3-p_4)\delta(p_1^-+p_2^--p_3^--p_4^-) \times\frac{(m_0\omega)^{d-1}}{2\omega}\int_0^x \mathrm{d}\eta\eta^{(-q_\mu^2/(8m_0\omega))+(d-3)/2}(1-\eta)^{-d+1} \sim \delta^{(d)}(p_1+p_2-p_3-p_4)\delta(p_1^-+p_2^--p_3^--p_4^-) \times\frac{(m_0\omega)^{d-1}}{2\omega}B_x\left(\frac{-q_\mu^2}{8m_0\omega}+\frac{d-1}{2}, -d+2\right), \quad (6.11)$$

where $1 \sim x = \mathrm{e}^{-2\omega\epsilon}$ and B_x is the incomplete Beta function. The longitudinal momentum conservation trivially is satisfied. Furthermore, because of the conservation of this momentum we do not expect so-called t-channel processes here.

6.1 Polology

Equivalently one may use the other representation of $K_{\text{oscillator}}$ as

$$K_{\text{oscillator}}(Y_\perp = 0, T_2; Y_\perp = 0, T_1) = \sum_n \langle 0|n\rangle\langle n|0\rangle\mathrm{e}^{-\mathrm{i}E_n(T_2-T_1)}, \quad (6.12)$$

with the E_n's as the known $H_{\text{oscillator}}$ eigenvalues. In this representation one finds the pole expansion [19]:

$$\langle p_3^\mu, p_4^\mu; \infty|p_1^\mu, p_2^\mu; -\infty\rangle \sim \delta^{(d)}(p_1+p_2-p_3-p_4)\delta(p_1^-+p_2^--p_3^--p_4^-) \times \lim_{\epsilon\to 0^+}\sum_n C_n\frac{\mathrm{i}4m_0}{q_\mu q^\mu - M_n^2 + \mathrm{i}\epsilon}. \quad (6.13)$$

This pole expansion also can be derived by extracting the poles of the amplitude (6.11) with the condition

$$\frac{-q_\mu^2}{8m_0\omega} + \frac{d-1}{2} = -n, \quad (6.14)$$

with n being a positive integer. Hence for the mass of the intermediate bound states we obtain

$$M_n^2 = \frac{8k\left(n+\dfrac{d-1}{2}\right)}{(g_\mathrm{s}^{1/3}l_\mathrm{s})^2}. \quad (6.15)$$

We recall that the combination $g_\mathrm{s}^{1/3}l_\mathrm{s}$ is l_{d+2}, the fundamental length of $d+2$ dimensional theory (Sect. 3 and [19]). The Regge pole expansion of (6.11)–(6.15) is the phenomenological promising feature of this amplitude [20–25].

[13] This cut-off is for extracting the contribution of graphs with a four-legs vertex, as $\lambda\phi^4$. From the time-energy uncertainty relation, we learn that these graphs are generated by super-heavy intermediate states

7 Discussion

In this section we discuss some relevant issues: (1) the large-N limit, (2) quark, gauge theory and gravity and the relations between the solutions, and also (3) non commutativity.

7.1 Large N

Baryons show special properties in the large-N limit of gauge theories [43].

(1) Their masses grow linearly with N.
(2) Their sizes do not depend on N, so their density goes to infinity at large N.
(3) The baryon–baryon force grows proportionally to N.

These properties mainly are extracted from a Hamiltonian formulation for the baryons as a bound-state of N quarks. Based on the approximation to approach the N-body problem (the Hartree approximation), the above properties can be justified for baryons at large N.

Here we try to work out the Hamiltonian formulation, and then the above mentioned properties are followed by the same reasoning as of [43][14].

In Sect. 4 the effective theory for D0-branes was found to be

$$S = \int \mathrm{d}t \left(\frac{1}{2} m_0 \sum_{(a)=1}^{N} \dot{\boldsymbol{r}}_{(a)}^2 - 4\pi \left(\frac{d-1}{2} \right) \sum_{(a)>(b)=1}^{N} \frac{|\boldsymbol{r}_{(a)} - \boldsymbol{r}_{(b)}|}{2\pi\alpha'} \right). \tag{7.1}$$

Also we have found the relation between the α' parameter and the coupling constant of gauge theory by comparing it to LClgt, namely $1/\alpha' \sim g_{\mathrm{YM}}^2/a^2$ where a is the lattice spacing parameter. It is known that it is more convenient to replace the coupling constant by $g_{\mathrm{YM}}/N^{1/2}$ at large N [43]. So the action in terms of the new parameters is

$$S = \int \mathrm{d}t \left(\frac{1}{2} m_0 \sum_{(a)=1}^{N} \dot{\boldsymbol{r}}_{(a)}^2 - 4\pi \left(\frac{d-1}{2} \right) \frac{g_{\mathrm{YM}}^2}{a^2} \frac{1}{N} \sum_{(a),(b)=1}^{N} |\boldsymbol{r}_{(a)} - \boldsymbol{r}_{(b)}| \right), \tag{7.2}$$

and the associated Hamiltonian is the same as used in [43] except for the potential term, which is a Coulomb one there.

Here we just check the mass of baryons at large N. The kinetic term of c.m., $\boldsymbol{P}^2/(Nm_0)$, grows with N, and the net potential for each D0-brane takes a factor $(1/2)N(N-1)$ due to pair interactions. So the potential term at large N grows like

$$\frac{1}{2} N(N-1) \frac{g_{\mathrm{YM}}^2}{N} \sim N. \tag{7.3}$$

We find as a result that the energy grows as $E \sim N$ at large N. From the point of view of the light cone frame the energy is P^-. The total longitudinal momentum of this bound state is $P^+ = Np^+$, where $p^+ = m_0$ is the longitudinal momentum of one D0-brane. Consequently, the invariant mass M is

$$M^2 = 2P^+P^- - \boldsymbol{P}^2 \sim N^2 \Rightarrow M \sim N. \tag{7.4}$$

7.2 Quarks, gauge theory and Schwartzschild solutions of gravity

Dp-branes are p dimensional Schwartzschild solutions of low energy effective field theories of string theories[15]. So any proposal for equivalence between them and quarks, or at least between their dynamics and quarks dynamics, may need justification at first. Here we recall some string theoretic related issues briefly, and also try to present a possible non-string theoretic related feature.

As mentioned, D-branes are gravity solutions. On the other hand, it is known that the dynamics of these objects is captured by a gauge theory. It is one of the closest connections between gauge theories and gravity which has been revealed by string theory. Based on this relation between the dynamics of an extended object and a gauge theory, many studies have been done to develop an understanding of the dynamics of gauge theory. One of the recent steps forward in this area is the adS/CFT correspondence [6], to relate gauge theory dynamics at large 't Hooft coupling ($\lambda = g_{\mathrm{YM}}^2 N$) to gravity in the anti-de Sitter background.

The relation between gauge theory and gravity is also studied at the level of the equations of motion. Both gravity and non-Abelian gauge theories, though in different orders, have non-linear equations of motion. It has been discovered that both pure gauge theories and gauge theories with matter have Schwartzschild-like solutions [44–47]. By Schwartzschild-like we mean the similarity between "connections" in gauge theories (known as gauge fields $A^{\mu}_{(a)}$) and gravity (known as connection coefficients $\Gamma^{\alpha}_{\beta\gamma}$). In the case of $SU(2)$ gauge theory with massless scalar matter fields the solution is found to be [45]

$$\begin{aligned} A_i^{(a)} &= \epsilon_{(a)ij} \frac{r^j}{g_{\mathrm{YM}} r^2} [1 - K(r)], \\ A_0^{(a)} &= \frac{r^{(a)}}{g_{\mathrm{YM}} r^2} J(r), \\ \phi^{(a)} &= \frac{r^{(a)}}{g_{\mathrm{YM}} r^2} H(r), \end{aligned} \tag{7.5}$$

[14] Because we have considered D0-branes in the light cone frame, for $p^+ = m_0 \gg l_{\mathrm{s}}^{-1}$, the heavy quark theory of [43] is a good approximation for the transverse dynamics of the D0-branes

[15] In superstring theories, they are charged solutions under a $p+1$-form field

with

$$K(r)=\frac{Cr}{1-Cr},\quad J(r)=\frac{B}{1-Cr},$$
$$H(r)=\frac{A}{1-Cr},\tag{7.6}$$

with $A^2-B^2=1$. The gauge field behavior is comparable with the connection coefficients in a Schwartzschild solution:

$$\Gamma^t_{rt}=\frac{K}{2r}\frac{1}{r-K},\quad \Gamma^r_{rr}=-\frac{K}{2r}\frac{1}{r-K},\tag{7.7}$$

with $K=2GM$. Here we just review some properties of the solution (7.5) [45]. First, both the gauge and scalar fields are singular at the radius $r_0=C^{-1}$. Further, by calculating electric and magnetic fields one sees that both are singular at r_0. Therefore a particle which carries an $SU(2)$ charge becomes confined if it enters the region $r<r_0$. The singularity of the field strengths at r_0 here is different from that of the gravitational Schwartzschild solution, which can be removed by a suitable choice of coordinates. Based on this picture of confinement of a charge in the region $r<r_0$, a model for the confinement of gauge theories has been presented in [47].

Also the solution has a monopole magnetic charge. This can be seen from the generalized 't Hooft field strength:

$$\mathcal{F}_{\mu\nu}=\partial_\mu(\hat\phi^{(a)}W^{(a)}_\nu)-\partial_\nu(\hat\phi^{(a)}W^{(a)}_\mu)$$
$$-\frac{1}{g_{\rm YM}}\epsilon^{(a)(b)(c)}\hat\phi^{(a)}(\partial_\mu\hat\phi^{(b)})(\partial_\nu\hat\phi^{(c)}),\tag{7.8}$$

with $\hat\phi^{(a)}\equiv\phi^{(a)}(\phi^{(b)}\phi^{(b)})^{-1/2}$. Hence, for the magnetic field we find

$$\mathcal{B}_i=\frac{1}{2}\epsilon_{ijk}\mathcal{F}_{ij}=-\frac{r^i}{g_{\rm YM}r^3},\tag{7.9}$$

which is the magnetic field of a point monopole with charge $-4\pi/g_{\rm YM}$. One can also find the electric field:

$$\mathcal{E}_i=-\mathcal{F}_{0i}=\frac{r_i}{g_{\rm YM}r}\frac{\mathrm{d}}{\mathrm{d}r}\frac{J(r)}{r}=\frac{B(2Cr-1)r_i}{g_{\rm YM}r^3(1-Cr)^2},\tag{7.10}$$

which at $r\to\infty$ does not show the behavior of Prasad–Sommerfield's solution ($1/r^2$), and the interpretation of a net electric charge near the origin is impossible. So this solution seems more like a magnetic monopole, and its relation to a "quark" (a source of an electric field) is out of reach; but here the idea of Mantonen–Olive duality, which changes the role of solitonic solutions for the fundamental objects seems relevant.

7.3 Why non-commutativity?

One of the most interesting aspects of D-branes is the non-commutativity of their relative coordinates. If the model of this paper has some relation with nature, the question will be raised of a possible justification for this non-commutativity. To resolve this question one may consider the following prescription: *The structure of space-time has to be in correspondence and in consistence with the propagation of fields.* In this way one finds the space-time coordinates as a 4-vector X_μ which behaves like an electromagnetic field 4-vector A_μ (spin 1) under boost transformations. This is just the same idea as in special relativity: to change the concept of space-time to be consistent with the Maxwell equations.

Also in this way supersymmetry is a natural continuation of the special relativity program: adding the spin 1/2 sector to the coordinates of space-time, as the representative of the fermions of nature. This leads one to the space-time formulation of the supersymmetric theories, and in the same way fermions are introduced into the bosonic string theory.

Now, what may be modified if nature has non-Abelian (non-commutative) gauge fields? In the present view of nature non-Abelian gauge fields cannot make spatially long coherent states; they are confined or too heavy. But the picture may be changed inside a hadron. In fact, recent developments of string theories voice this change and it is understood that non-commutative coordinates and non-Abelian gauge fields are two sides of one coin. As we discussed, the interaction between D-branes is the result of path integrations over fluctuations of the non-commutative parts of coordinates. This means that in this picture "non-commutative" fluctuations of space-time are the source of "non-Abelian" interactions. This picture may justify involving the non-commutative coordinates in the study of bound states of quarks and gluons. One may summarize this idea as in Table 1.

Table 1. Non-commutative coordinates in the study of bound states of quarks and gluons

Field	Space-time coordinate	Theory
Photon A^μ	X^μ	electrodynamics (and QED)
Fermion ψ	$\theta,\bar\theta$	supersymmetric
Gluon $A^\mu_{(a)}$	$X^\mu_{(a)}$	chromodymamics (and QCD)

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A Particle electrodynamics in light cone frame

We just follow the steps of [30] in going to the light cone frame. The classical action is

$$S = -m\int_1^2 \mathrm{d}\tau\sqrt{\dot{x}^2} + q\int A_\mu \dot{x}^\mu \mathrm{d}\tau, \tag{A.1}$$

with the momentum

$$p^\mu \equiv -\frac{\partial L}{\partial \dot{x}_\mu} = m\frac{\dot{x}^\mu}{\sqrt{\dot{x}^2}} - qA^\mu. \tag{A.2}$$

Consequently one finds the constraints for the momenta and canonical Hamiltonian

$$(p^\mu + qA^\mu)(p_\mu + qA_\mu) = m^2, \tag{A.3}$$
$$H_\mathrm{c} = -p_\mu \dot{x}^\mu - L \equiv 0. \tag{A.4}$$

The total Hamiltonian will be found to be

$$H_t = \lambda((p^\mu + qA^\mu)^2 - m^2), \tag{A.5}$$

where λ is a Lagrange multiplier, and we have the canonical Poisson bracket $\{x^\mu, p^\nu\} = -\eta^{\mu\nu}$. So one finds that the dynamics has gauge symmetry (reparameterization invariance), and to find the Lagrange multiplier one should fix the gauge by the condition $\chi(x;\tau) \equiv 0$. Preserving gauge fixing in time gives

$$\dot{\chi} = 0 = \frac{\partial\chi}{\partial\tau} + \{\chi, H_t\}, \tag{A.6}$$

which gives

$$\lambda = -\{\chi, \theta\}^{-1}\frac{\partial\chi}{\partial\tau}, \quad \theta \equiv (p^\mu + qA^\mu)^2 - m^2. \tag{A.7}$$

The light cone gauge fixing is $\chi = \tau - x^+ = 0$, and also by adding the gauge fixing for the gauge field by $A^+ = 0$ [29], one finds for the momentum conjugate of the time (x^+), i.e. the Hamiltonian:

$$H = p^- = \frac{(\boldsymbol{p} + q\boldsymbol{A})^2}{2p^+} - qA^-, \tag{A.8}$$

where we have assumed $m = 0$. By taking p^+ as the Newtonian mass m_0 in the transverse directions and A^- as A_0, one gets the Lorentz equation of motion (5.1) for this Hamiltonian.

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